Unit 4: Series and parallel connections

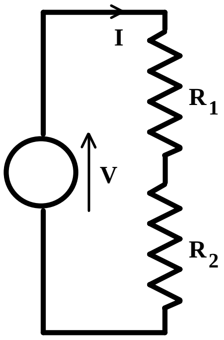


Fig. 4.1 Series connection of two resistances

The analysis of a circuit can be simplified by reducing the effective number of components present in the circuit. There are simple rules for combining resistances connected in series or parallel to a single equivalent resistance. These rules are easy to remember and to apply, but mistakes can occur if series and parallel connections are not correctly identified. Therefore, in this Unit we will adopt a rigorous approach, using the basic circuit laws to identify series and parallel connections and to derive rules for simplification.

Fig. 4.1 shows two resistances, R_1 and R_2, connected in a circuit with a single voltage source, V. A current, I, flows from the voltage source. The circuit would be simplified if we could find a single resistance, R, which could be connected to the voltage source and would cause the same current to flow from the source. How is the value of the equivalent resistance, R, related to the original pair of resistances, R_1 and R_2 ?

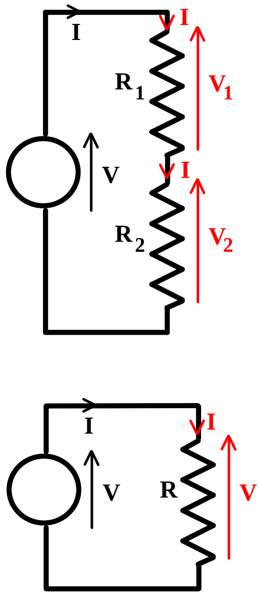


Fig. 4.2 Simplification of a series connection to a single equivalent, R

Fig. 4.2 shows the original connection of two resistances and its single resistance equivalent. The current I flows from the voltage source and through the closed circuit formed by the two resistances. Because the same current is flowing through all three circuit elements, the circuit is commonly referred to as a 'series circuit' and the two resistances are said to be *'connected in series'*. The strict definition of a series connection, therefore, is that series-connected elements carry the same current. All of the following analysis is valid only if we can be certain that the two resistances are carrying the same current. The voltages V_1 and V_2 in Fig. 4.2 are related to the current I by Ohm's Law:

$$V_1 = R_1 I \text{ and } V_2 = R_2 I \qquad (4.1)$$

and applying Kirchhoff's Voltage Law around the closed circuit loop:

$$+V_1 + V_2 - V = 0 \quad \text{or} \quad V = V_1 + V_2 \qquad (4.2)$$

Substituting from Eqn. 4.1 into Eqn. 4.2:

$$V = R_1 I + R_2 I = \{R_1 + R_2\}\, I \qquad (4.3)$$

For the equivalent single resistance circuit, shown on the right in Fig. 4.2, the source voltage V is equal to the voltage across the resistance R (from Kirchhoff's Voltage Law), so:

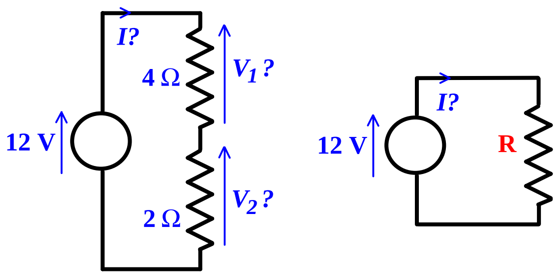
$$V = R\, I \qquad (4.4)$$

Comparing Eqns. 4.3 and 4.4, we see that the same relationship between voltage and current exists in both circuits if:

$$R = \{R_1 + R_2\} \qquad (4.5)$$

Worked example 4.1

Calculate the current *I* flowing from the 12 V source and the voltages, V_1 and V_2, across the two resistances.



Solution

From the circuit diagram, it is clear that the current *I* flows from the source and through both resistances, so the resistances are series-connected. Eqn. 4.5 can be applied to find the single equivalent resistance:

$$R = 4 + 2\ \Omega = 6\ \Omega.$$

The voltage across this resistance is equal to the source voltage of 12 V, so the current, $I = 12 / 6$ A = 2 A.

The same current of 2 A flows through the resistances in the original circuit and applying Ohm's Law:

$V_1 = 4 \times 2$ V = 8 V and $V_2 = 2 \times 2$ V = 4 V

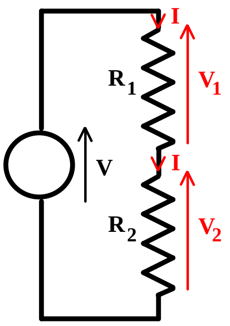


Fig. 4.3 Potential divider

Looking back at the results in Worked Example 4.1, we see that the source voltage of 12 V is divided between the two resistances in proportion to the resistance values (the 4 Ω resistance has a voltage of 8 V and the 2 Ω resistance has a voltage of 4 V). This result is an illustration of the *'potential divider'* rule, which applies to series-connected resistances. Fig. 4.3 shows the general case for a potential divider with two resistances. From Eqn. 4.5 the two individual resistances have the same effect on the current flowing around the circuit as a single resistance:

$$R = \{R_1 + R_2\} \text{ , so } I = V / \{R_1 + R_2\}$$

and applying Ohm's Law to the two resistances separately:

$$V_1 = R_1 I = V\, R_1/\{R_1 + R_2\} \text{ and } V_2 = R_2 I = V\, R_2/\{R_1 + R_2\} \qquad (4.6)$$

The potential divider rule must be used with great caution: it can be applied only in situations where resistances are series-connected in the strict sense that the same current is flowing through both resistances.

The ideas of circuit simplification by combining series-connected resistances and of the potential divider can be extended to connections involving larger numbers of resistances. The approach used in deriving Eqns. 4.5 and 4.6 can be extended to the general case of n resistances, shown in Fig. 4.4, where:

$$R = \{R_1 +....+ R_k +....+ R_n\} \qquad (4.7) \qquad \text{and:} \qquad V_k = V\, R_k/\{R_1 +....+ R_k +....+ R_n\} \qquad (4.8)$$

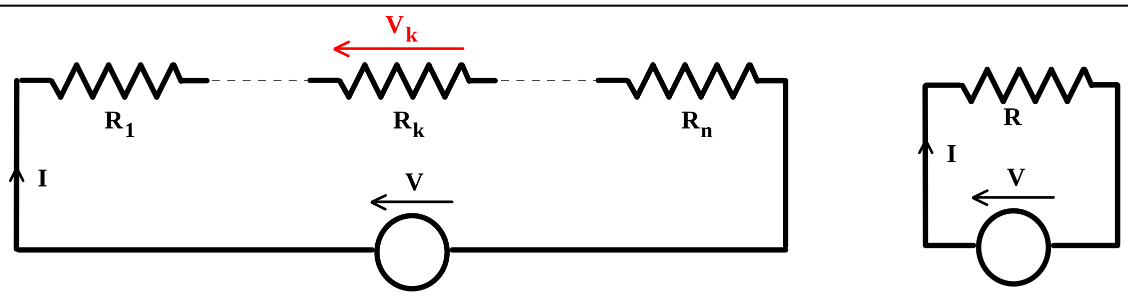
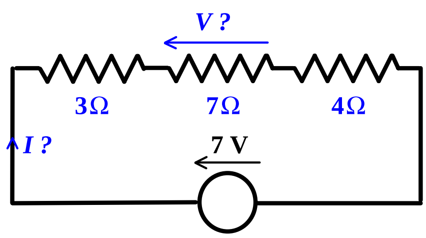


Fig. 4.4 Series connection of n resistances

Worked example 4.2

Calculate the current *I* and voltage *V*..



Solution

The three resistances are connected in series, so the single equivalent resistance (from Eqn. 4.7) is:

R = 3 + 7 + 4 Ω = 14 Ω

and applying Ohm's Law: *I* = 7 / 14 A = 0.5 A

The voltage *V* can be found by applying the potential divider rule (Eqn. 4.8):

V = 7 x 7 / {3 + 7 + 4} V = 3.5 V

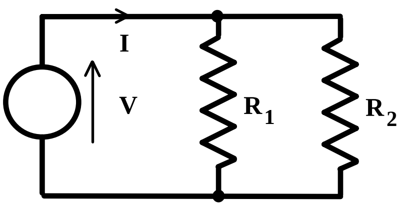


Fig. 4.5 Parallel connection of two resistances

Parallel connection is illustrated in Fig. 4.5, where two resistances, R_1 and R_2, are connected in a circuit with a single voltage source, V. A current, I, flows from the voltage source. It would be useful to find, in terms of R_1 and R_2, the value of a single resistance, R, which could be connected to the voltage source and would cause the same current to flow from the source. It will be apparent that the voltage across each of the circuit elements is equal to the source voltage, V (for a proof of this result, Kirchoff's Voltage Law can be applied to the two circuit loops). The same voltage is present across the terminals of the two resistances, and therefore the resistances are said to be 'connected in parallel'. {Note the comparison with a series connection, where it is the *current* which is common to both resistances}. The following derivation of the relationship between two parallel-connected resistances, R_1 and R_2, and a single equivalent resistance, R, is valid only if the two resistances have a common terminal voltage.

Referring to the voltage and current variables defined in Fig. 4.6, the currents, I_1 and I_2, flowing through the two original resistances are related to the common voltage, V, by Ohm's Law:

$I_1 = V / R_1$ and $I_2 = V / R_2$ (4.9)

and applying Kirchhoff's Current Law at a node:

$+ I_1 + I_2 - I = 0$ or $I = I_1 + I_2$ (4.10)

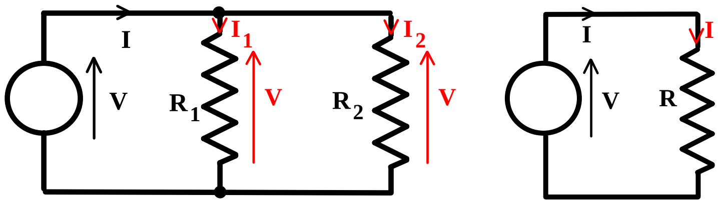


Fig. 4.6 Simplification of a parallel connection to a single equivalent, R

Substituting from Eqn. 4.9 into Eqn. 4.10:

$I = (V / R_1) + (V / R_2) = V \{1/R_1 + 1/R_2\}$ (4.11)

For the equivalent single resistance circuit, shown on the right in Fig. 4.6, the source voltage V is equal to the voltage across the resistance R (from Kirchhoff's Voltage Law), so the current I flowing from the source and through the resistance is::

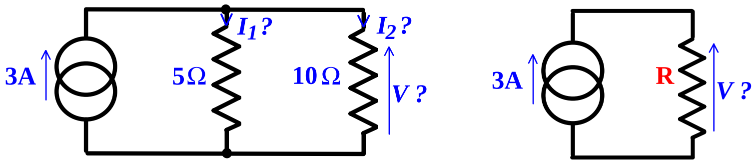
$I = V / R$ (4.12)

Comparing Eqns. 4.11 and 4.12, we see that the relationship between current and voltage is the same in both circuits if:

$1/R = \{1/R_1 + 1/R_2\}$, which can be re-arranged: $R = R_1R_2 / \{R_1 + R_2\}$ (4.13)

Worked example 4.3

Calculate the voltage *V* and the currents, I_1 and I_2, through the two resistances.



Solution

From the circuit diagram, it is clear that the two resistances are in parallel. Eqn. 4.13 can be applied to find the single equivalent resistance:

$R = 10 \, x \, 5 / \{10 + 5\} \, \Omega = 3.33 \, \Omega$.

{It's easy to make a mistake when using Eqn. 4.13, but a simple check is to remember that the single resistance must have a value smaller than the smallest of the separate resistances. Here 3.33 Ω is smaller than 5 Ω, so the answer is probably correct}

The voltage across this resistance can be calculated using Ohm's Law, $V = 3.33 \, x \, 3$ V = <u>10 V</u>

The voltage *V* is applied to the two separate resistances in the original circuit and applying Ohm's Law:

$I_1 = 10 / 5$ A = <u>2 A</u> and $I_2 = 10 / 10$ A = <u>1 A</u>

When dealing with parallel connections, it is often convenient to work with the reciprocal of resistance:

conductance, $G = 1 / R$

The SI units of conductance are Siemens (S). Note that the unit of Siemens is abbreviated to a capital letter S, to avoid confusion with the abbreviated units of time, seconds (s). Ohm's Law can be re-written for conductance:

$I = G V$

Combining separate parallel conductances to a single equivalent (Fig. 4.7) is particularly simple:

$I_1 = G_1 V$ and $I_2 = G_2 V$ (4.14)

Applying Kirchhoff's Current Law at a node:

$+ I_1 + I_2 - I = 0$ or $I = I_1 + I_2$ (4.15)

Substituting from Eqn. 4.14 into Eqn. 4.15:

$I = G_1 V + G_2 V = V \{G_1 + G_2\}$ (4.16)

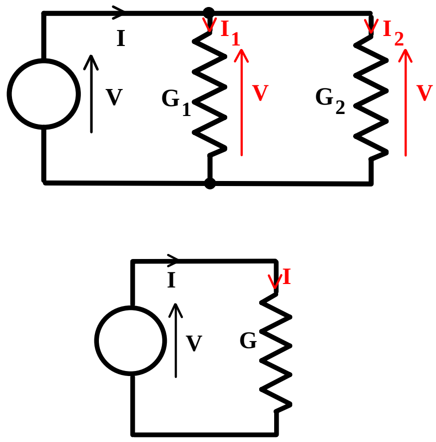


Fig. 4.7 Simplification of a parallel connection to a single equivalent, G

For the equivalent single resistance circuit, shown on the right in

Fig. 4.7, the source voltage V is equal to the voltage across the conductance G, so the current I flowing from the source and through the resistance is::

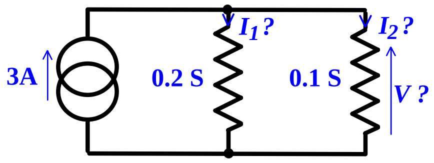
$$I = G V \qquad (4.17)$$

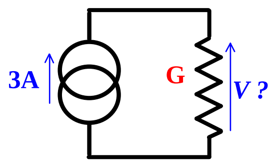
Comparing Eqns. 4.16 and 4.17, the relationship between current and voltage is the same if:

$$G = G_1 + G_2 \qquad (4.18)$$

Worked example 4.4

Calculate the voltage *V* and the currents, I_1 and I_2, through the two conductances. *{This problem is the same as Worked Example 4.3, except that the resistances in that problem have been converted to conductances.}*





Solution

The two conductances are in parallel and can be combined to a single equivalent conductance, using Eqn. 4.18:

$G = 0.2 + 0.1$ S $= 0.3$ S.

The voltage across this single conductance is calculated using Ohm's Law, $V = I / G = 3 / 0.3$ V = 10 V

The voltage *V* is applied to the two separate conductances in the original circuit and applying Ohm's Law:

$I_1 = 0.2 \times 10$ A = 2 A and $I_2 = 0.1 \times 10$ A = 1 A

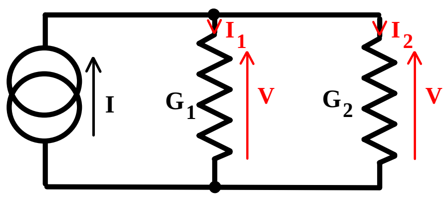


Fig. 4.8 Current divider

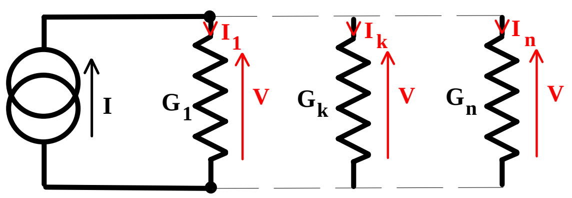


Fig. 4.9 Parallel connection of n conductances

If we examine the solution to the last example carefully, we observe that the source current of 3 A divides between the conductances in proportion to their value, with 2 A flowing through the conductance of 0.2 S and 1 A flowing through 0.1 S. The *'current divider'* rule applies to parallel-connected conductances. Fig. 4.8 shows the general case for a current divider with two conductances. From Eqn. 4.18 the two individual conductances have the same effect as a single conductance:

$$G = \{G_1 + G_2\} \text{ , so } V = I / \{G_1 + G_2\}$$

and applying Ohm's Law to the two conductances:

$$I_1 = G_1 V = I\, G_1/\{G_1 + G_2\}$$

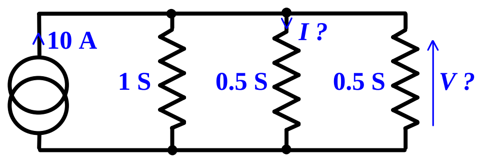
$$\text{and } I_2 = G_2 I = I\, G_2/\{G_1 + G_2\} \qquad (4.19)$$

Circuit simplification by combining parallel-connected conductances and current divider rule can be extended to connections involving larger numbers of conductances. The derivation of Eqns. 4.18 and 4.19 can be extended to the general case of n conductances, shown in Fig. 4.9, where:

$$G = \{G_1 + \ldots + G_k + \ldots + G_n\} \qquad (4.20) \quad \text{and:} \quad I_k = I\, G_k/\{G_1 + \ldots + G_k + \ldots + G_n\} \qquad (4.21)$$

Worked example 4.5

Calculate the voltage, *V*, and current *I*..



Solution

The three conductances are connected in parallel, so the single equivalent conductance (from Eqn. 4.20) is:

$G = 1 + 0.5 + 0.5$ S $= 2$ S

and applying Ohm's Law: $V = I / G = 10 / 2$ V = 5 V

The current *I* can be found by applying the current divider rule (Eqn. 4.21):

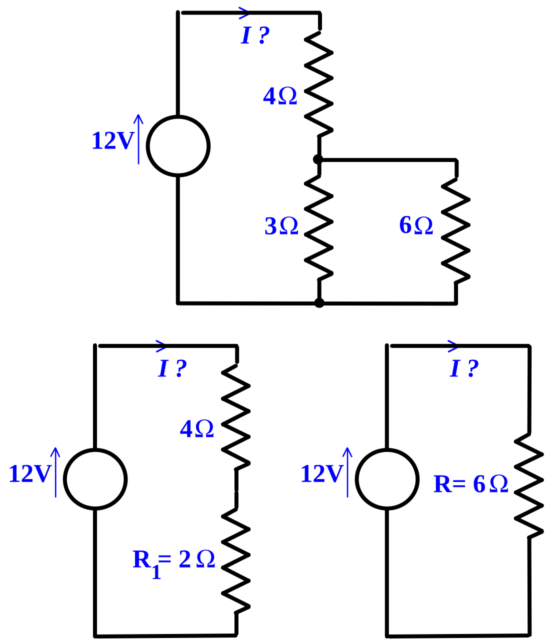
$I = 10 \times 0.5 / \{1 + 0.5 + 0.5\}$ A = 2.5 A

Unit 4: Series and parallel connections

Many circuits involve a combination of series and parallel connections. When attempting to simplify more complex circuits take great care in identifying components that are connected in series or parallel. Remember that components are in series only if it is certain that the same current flows through them, while parallel-connected components have a common voltage at their terminals. The following examples illustrate the simplification of more complicated circuits.

Worked example 4.6

Calculate the current, *I*..



Solution

The connection of three resistances can be reduced to a single equivalent resistance connected to the voltage source in two stages, as illustrated in the diagram. The 3 Ω and 6 Ω resistances are connected in parallel and can be reduced to a single resistance:

$R_1 = 3 \, x \, 6 \, / \, \{3 + 6\} \; \Omega \; = 2 \; \Omega.$

This resistance is in series with the 4 Ω resistance, so the total circuit resistance is:

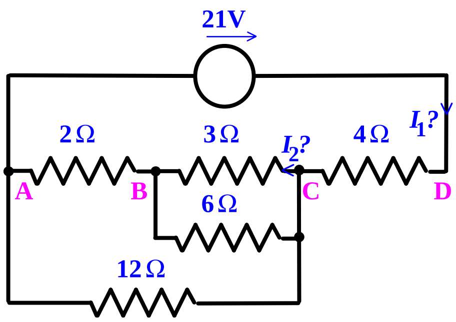
$R = 4 + 2 \; \Omega \; = 6 \; \Omega.$

and therefore the current flowing from the 12 V source, by Ohm's Law, is:

$I = 12 / 6 =$ <u>2 A</u>

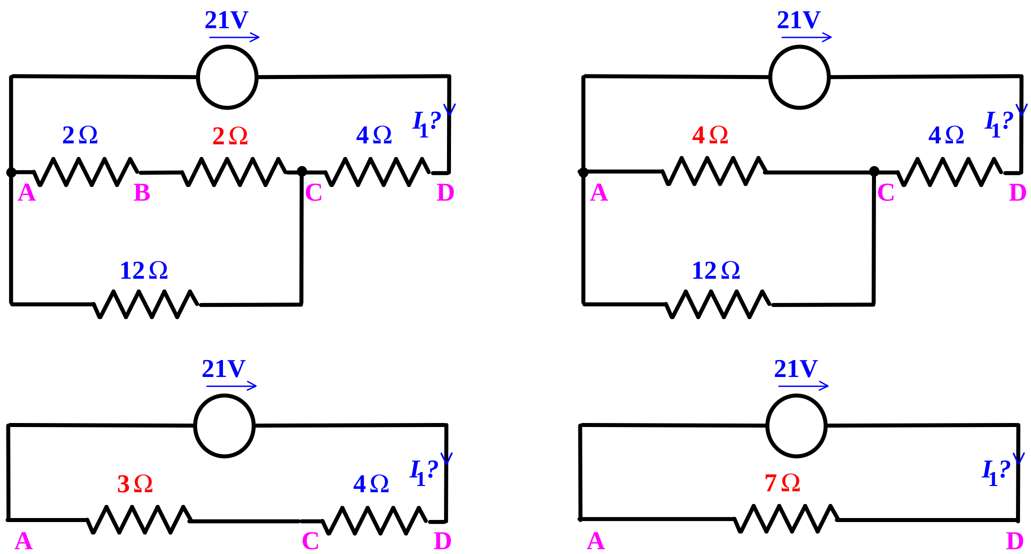
Worked example 4.7

Calculate the currents, I_1 and I_2..



Solution

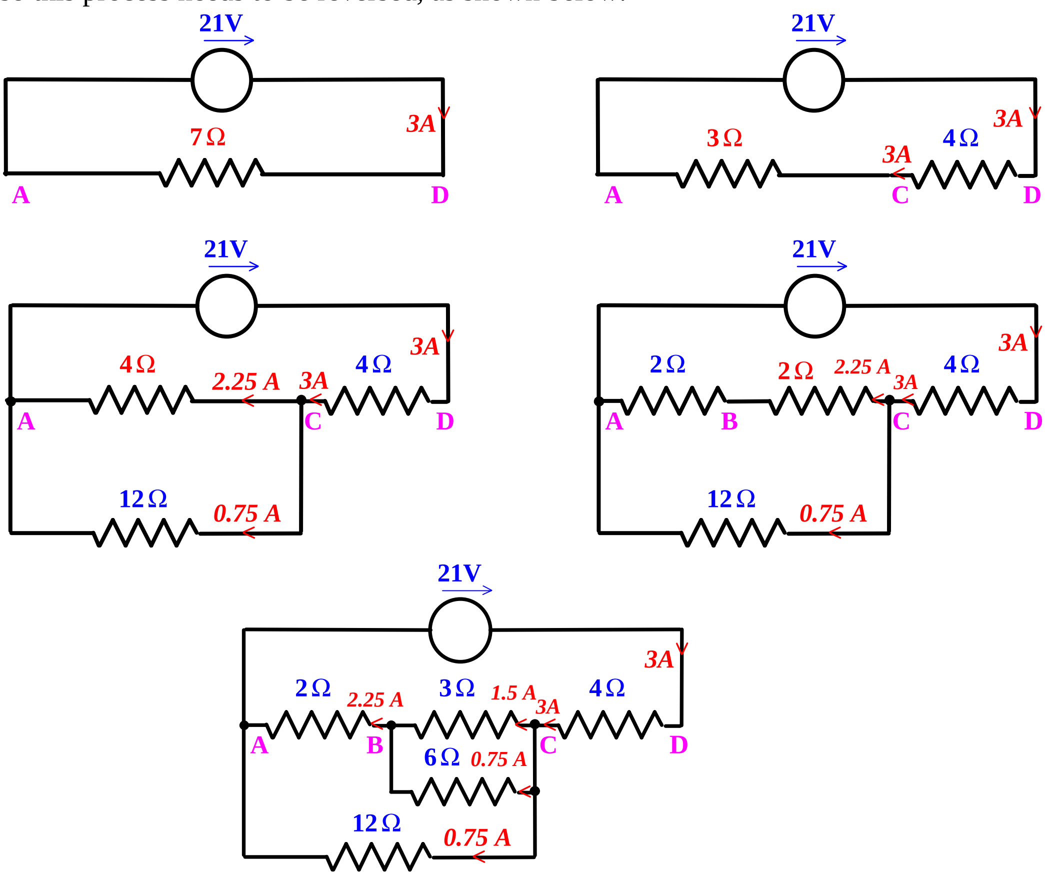
In a more complicated problem it is a good idea to label key points in the circuit, so that errors can be avoided when redrawing the circuit layout. Here 4 key points A, B, C, D have been labelled. Looking carefully at the diagram, we see that there is only one immediate possibility for simplification: the two resistances (3 Ω and 6 Ω) between points b and C are connected in parallel and can be reduced to a single resistance of 2 Ω. The circuit can be redrawn as in the diagram below.



Now the circuit can be simplified further, because there are two resistances (each 2 Ω) connected in series between points A and C. These two resistances can be combined in series to an equivalent of 4 Ω, which is in parallel with the 12 Ω resistance also connected between A and C. Therefore the total effective resistance between A and C is equal to 3 Ω. Finally, we see that the circuit between A and D comprises just two series-connected resistances (3 Ω and 4 Ω), which are equivalent to a single 7 Ω resistance.

The calculation of the current I_1 is straightforward, because the 21 V source is applied to the total circuit resistance of 7 Ω, causing a current of 3 A to flow into the circuit. Hence $\underline{I_1 = 3\text{ A}}$.

Calculation of I_2 is a little more difficult. The current has 'lost' during the circuit simplification process, so this process needs to be reversed, as shown below.



The current of 3 A flows through the 4 Ω resistance connected between points D and C, because this circuit element is connected in series with the 21 V source. At point C the 3 A current splits and the current divider rule can be used to show that 0.75 A flows through the 12 Ω resistance connected directly between c and A in the original circuit, while 2.25 A flows through the 4 Ω resistance, which is an equivalent of all other paths between C and A. {You can check that this current division is correct by observing that the voltage of C relative to A is 9 V.} The resistance of 4 Ω between C and A is built up from a series connection of the 2 Ω resistance between B and A, together with the equivalent resistance of 2 Ω representing the circuit connected between C and B. The current of 2.25 A flows through both of these resistances. Concentrating now on the section of circuit between C and B, we see that in the original circuit it consists of two parallel resistances (3 Ω and 6 Ω), through which flows a total current of 2.25 A. The current divider rule can be used to show that the current flowing through the 3 Ω resistance is 1.5 A and therefore: $\underline{I_2 = 1.5\text{ A}}$.

It would be a mistake to think that any circuit can be simplified by combining elements connected in series and parallel. For example, consider the circuit shown in Fig. 4.10. Here none of the resistances can be combined, and yet the current flowing from the voltage source must be defined by an 'equivalent' circuit

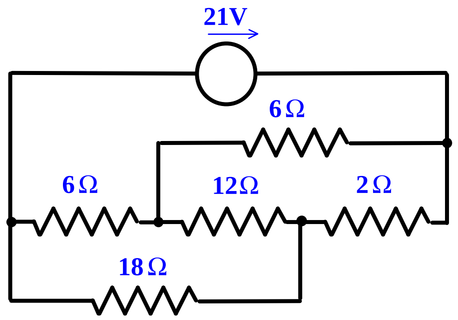


Fig. 4.10 A circuit which cannot be simplified by series / parallel combination

resistance. How can this resistance be calculated from the circuit resistances? For a solution to this problem we must wait until we learn about star / delta and delta / star transformations in a future Unit.